

Introduction to Computer Algorithms CS470
Second Midterm
Thursday, April 7, 2005
12:30pm – 1:45pm

Student name

Student number

For full credit solve problem 1. You can use two letter-size pages of your own notes, written on any of the two sides of the page; no other material is permitted (e.g., no books).

Problem 1: [70 points]

Imagine that you are a manager of a company. You have a collection of N items. Each item i has size s_i and value v_i , such that s_i and v_i are positive integers (>0). These items are located in a storage facility in Tuscaloosa. You have a car with a trunk of size $S > 0$, also an integer. If you transport items to Birmingham, then you can sell them at their value. You can make only one trip. Therefore, you want to decide which items to put into the trunk so as to maximize the sum of values of the items in the trunk. Of course the sum of sizes of items that you put into the trunk cannot exceed the size of the trunk! Your goal is to design an algorithm that solves this maximization problem. The running time of your algorithm must be polynomial with respect to N and S (e.g., $O(N^5 \cdot S^3)$).

Example

$N=4, S=10$

item	size	value
1	5	20
2	4	10
3	6	18
4	2	4

Then it is best to put item 1 and 2 into the trunk, which yields the total value of 30.

Prove in detail why your algorithm is correct. Provide an asymptotic upper bound on its worst-case running time.

Solution: