Problem set number 8
CS470 Fall 2005
due date December 4, 2005, 11:59pm

Exercise 8.1 [worth 0 points]
Read lecture notes (http://www.cs.ua.edu/470/fall2005/Lec_08.pdf and http://www.cs.ua.edu/470/fall2005/Lec_09.pdf). The reading will help you solve the subsequent problems.

Exercise 8.2 [worth 10 points]
Consider the following graph G=(V, E), where V= {s, t, u, v, w, x, y, z} and E= {s-t, s-u, t-u, t-v, t-y, t-z, u-v, u-w, v-w, v-x, v-y, w-x, x-y, y-z} and the weights: {w(s-t)=1, w(s-u)=4, w(t-u)=2, w(t-v)=9, w(t-y)=4, w(t-z)=2, w(u-v)=1, w(u-w)=3, w(v-w)=1, w(v-x)=3, w(v-y)=1, w(w-x)=1, w(x-y)=6, w(y-z)=14 }. Trace the Dijkstra's algorithm used to compute a shortest path from x to every other node. Do this by completing the following table:

Iteration	*S*	*d[s]; P[s]*	*d[x]; P[x]*	*d[t]; P[t]*	*d[u]; P[u]*	*d[v]; P[v]*	*d[w]; P[w]*	*d[y]; P[y]*	*d[z]; P[z]*
0	{s}	0; s							
1	{s,t}	0; s		1; s,t					
…									

Exercise 8.3 [worth 20 points]
Suppose that you have a map of the railroad system of a country, connecting its cities. The country has a seaport and n cities. Some pairs of cities are connected by a direct railroad, some are not. Traveling along a direct railroad from a city A to a city B comes at a cost $c_{A\text{-}B} \geq 0$, the same as traveling in the opposite direction. You have a load of goods that can fill in exactly $k \leq n$ trains. Your goal is to select the city where each train will be sent to; where the goods will get unloaded. Sending a train along a path from the seaport to a city comes at the cost equal to the sum of costs of traveling along the railroads along the path. Naturally, you want to minimize the total cost of sending the trains to their destinations. Design an algorithm that takes as input the graph modeling the railroad and costs, and k, and produces the k paths for the k trains, that minimize the cost of sending the trains along the paths. Argue why there is no collection of k paths yielding strictly lower cost compared to the cost of the paths that your algorithm has returned.

Exercise 8.4 [worth 15 points]
Solve 26.3-3 from page 668 of CLRS.