Introduction to Computer Algorithms CS470
Second Midterm
Friday, November 18, 2005
9:00am – 10:00am

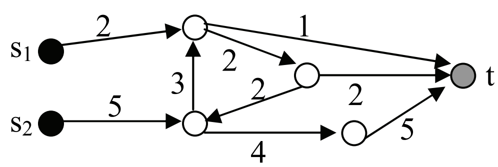
Student name ………………………………………………………

Student number ………………………

For full credit solve problems 1 through 3. You can use two letter-size pages of your own notes, written on any of the two sides of each page; no other material is permitted (e.g., no books).

Problem 1: [20 points]

Consider the following "extended" flow network G, where there is one sink t, but two sources s_1 and s_2; each edge had a capacity as depicted



Find a maximum flow from the sources s_1 and s_2 to the sink t (each source can "push" flow into the network). Explain how the Ford-Fulkerson algorithm can be used for this extended flow network. How would you use the algorithm with an arbitrary number of sources, not just two?

Solution

Problem 2: [15 points]

Suppose that you are presented with a map of a transportation network of a country. Certain cities are connected by direct roads, but certain are not. The network requires renovation. For each road there is a certain positive cost of restoring the road. However, some roads must not be renovated at all, since they will be closed. Design an algorithm for selecting, if possible, the a set roads for renovation other than these that must get closed, so as to minimize the total cost of renovation, while allowing the traffic to travel between any cities through renovated roads. When selection is not possible, your algorithm must report that. The transportation network is provided to you as an undirected graph G.

Solution

Problem 3: [20 points]

Let G be a directed graph. A ***transitive closure*** T(G) is a directed graph on the same nodes as G, such that there is an edge u→v in T(G) if, and only if, there is a path of length at least one from u to v in G. Prove the following lemma.

Lemma
Given a directed acyclic graph G and a linear ordering S of its nodes:
S is a topological sort G if, and only if, S is a topological sort of T(G)

Proof