

**Introduction to Computer Algorithms CS470**  
**Second Midterm**  
**Friday, November 12, 2004**  
**10:00am – 10:50am**

Student name .....

Student number .....

**For full credit solve problem 1. You can use a letter-size page of your own notes, written on any of the two sides of the page; no other material is permitted (e.g., no books).**

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**Problem 1: [60 points]**

Consider the following optimization problem

**Generalized assembly-line scheduling**

**Input:** the number  $m \geq 1$  of assembly lines, each consisting of  $n \geq 2$  stations, processing time  $a_{i,j} \geq 0$  at station  $S_{i,j}$ , for  $1 \leq i \leq m$ ,  $1 \leq j \leq n$ , transfer time  $t_{i,j,k} \geq 0$  from station  $S_{i,j}$  to station  $S_{k,j+1}$ , for  $1 \leq i, k \leq m$ ,  $1 \leq j \leq n-1$ , entry time  $e_i \geq 0$  from the starting point to station  $S_{i,1}$ , and exit time  $x_i \geq 0$  from station  $S_{i,n}$  to the exit point, for  $1 \leq i \leq m$ .

**Output:** A fastest route of chassis through the factory.

- a) Draw a picture representing the factory when  $m=3$  and  $n=5$  (indicate a few times, not all).
- b) State and prove a lemma describing optimal structure of the problem (use pictures to facilitate your proof).
- c) State a recurrence and prove that the recurrence is correct.
- d) Design an algorithm that finds a fastest route through the factory; its running time should be polynomial with respect to  $m$  and  $n$ .

**Solution:**