Introduction to Computer Algorithms CS470
Final Examination
Monday, December 13, 2004
11:30am – 2:00pm

Student name ………………………………………………………

Student number ………………………

There are three problems. For full credit solve all of them. You can use three letter-size pages of your own notes, written on any of the two sides of the pages; no other material is permitted (e.g., no books).

Problem 1: [25 points]

Imagine that there is a long road. Salesmen of certain companies sell their products along the road. Each salesman works within a certain segment of the road, starting at a place and ending at a place. If two salesmen work on a shared part of the road, then there exists competition between them. We would like to detect if competition occurs or not. We can abstract this problem as follows. A closed interval I can be represented as I=[a,b], where $a \le b$. Given a collection of n intervals $I_i=[a_i,b_i]$, for $1 \le i \le n$, we want to determine if there is $i<j$ such that interval I_i has non-empty intersection with interval I_j. If so, we say that there is ***competition***. Otherwise, we say that there is no competition. Give an O(n log n) algorithm for this problem. The running time of your algorithm should not depend on how big a and b are. Argue in detail why your algorithm is correct, and why its running time is as claimed. Specifically, prove the following theorems.

Theorem
If there is competition, then the algorithm reports "competition". If there is no competition, then the algorithm reports "no competition".

Theorem
The running time of the algorithm is O(n log n).

Solution:

Problem 2: [15 points]

Suppose that we are given n positive integers $a_1,\ldots,a_n$ and a positive "target" integer t. We want to find out whether or not there is a subsequence $i_1<\ldots<i_k$ such that $a_{i1}+\ldots+a_{ik}=t$. Give an O(nt) algorithm for this problem. Prove that your algorithm is correct.

Solution:

Problem 3: [15 points]

Given a directed graph G=(V,E) we say that a directed graph G'=(V,E') on the same set of vertices is a ***transitive closure*** of G when:

there is an edge (u,v) in G' if, and only if, there is a path of length at least one from u to v in G.

Design an algorithm for calculating a transitive closure of a directed graph whose running time is $O(|V|^3)$. Argue why your algorithm is correct.

Solution: